NJAAPT NEWSLETTER
November - 2009

President's Message

Welcome This year has been one that has provided new challenges for the Section and the executive board is working very hard to meet the needs of the membership. There have been a number of very uplifting signs displaying an enthusiasm that has been a characteristic of the New Jersey Section over the years. However, on the downside, there appears to be less interest in participating in sponsored workshops.

How do you acquire additional skills to make your class better? When asked this question, the response should easily be: "By attending workshops and meetings." The NJAAPT tries to provide these opportunities for you and spends much time and effort in planning. Why not take advantage of meeting with your fellow physics teachers to share and learn in a very relaxed and friendly setting?

Here's a brief summary of what has happened so far:

- A September workshop was cancelled due to lack of registrants
- A meeting with the Physics Club at Montclair State University was well received.
- Our participation at the NJSC in sponsoring the Demo Dens and workshops drew large numbers
- Registration for a November workshop is lagging
- Holiday Treats filled up in record time

Ray Polomski

New Jersey Science Convention 2009

If you The NJAAPT once again made its presence felt at the annual NJ Science Convention by sponsoring workshops and the Physics Demo Dens. Our table was manned for most of the two days by Jim Kovalcin, Joe Spaccavento, and Ray Polomski. Jim did a great job in providing the attendees with a panorama of the activities the NJAAPT sponsors during the year. Handouts of materials provided by the AAPT filled the bags of those stopping by the table. New members were signed up for both the section and the national organizations.

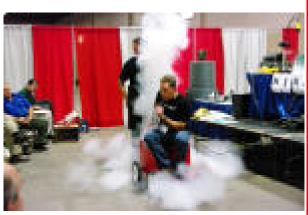
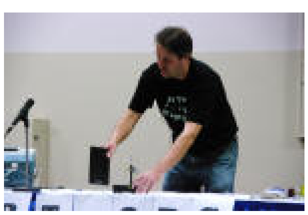
Demo Den #1 - Linear Kinematics



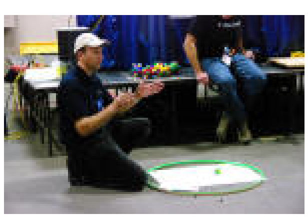
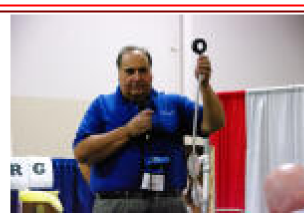
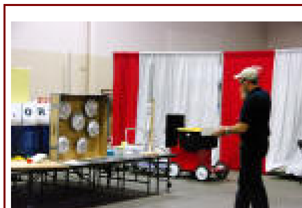

Demo Den #2 - Rotational Kinematics




Demo Den #3 - "A Demo a Day"

Demo Den #4 - Modern Physics & Astronomy

Go to the NJAAPT website for more pictures!

The Demo Dens organized by Borislaw Bilash entertained teachers from all levels of science teaching and covered a multitude of topics. Along with Dave Maiullo, Borislaw enlisted the aid of other members, too numerous to mention, to share demonstrations with the audience in a fast paced show that everyone enjoyed. Photos of the programs can be viewed on our website and for further information about a demo contact either Borislaw or Dave – they would be glad to help. Our thanks go out to all who donated their time and energy to make the Demo Dens a rousing success.

The dinners provided an opportunity for three of our members to be recognized for their contributions to education. On Tuesday evening, Jessie Blair, our secretary and past president, was presented with the Thomas Fangman Award. Jessie was honored for her many years of work at the NJSC.

The following evening, Dave Maiullo, was recognized by the dinner committee for having received the Contributor Award from the AAPT at the Summer Meeting in Ann Arbor, Michigan. John Johnston was given the NJAAPT Lifetime Contribution to Physics Education Plaque in recognition of his service to the NJAAPT. John has conducted many workshops, demonstration shows, and participated in sharing sessions and remains an integral part of the NJAAPT even after eighteen years of retirement.

About AAPT Membership

It is a common belief in our society that professional people should belong to their professional organization**.** The **American Association of Physics Teachers** has been the voice of a world-wide community of physics educators since 1931. Has the over $100 yearly dues forced you from joining yours! Would you change your mind for a "DIME" a day? Recently, due to a great deal of pressure from the High School Physics teaching community, the AAPT has reduced the "Regular Pre-College Membership" to $79 per year. Now, AAPT has created a new **"Associate Membership"** for individuals who have never joined AAPT and former members whose membership lapsed before December 31, 2006. Your Associate Member Benefits Include the following:

- ✓ Full-text access to 42 electronic articles from *The Physics Teacher* or the *American Journal of Physics*.
- ✓ Full access to Physics Today print and online.
- ✓ Receive the Association's electronic newsletter - the *eNNOUNCER*.
- ✓ Receive AAPT Physics Store coupons valued at over $200.
- ✓ Reduced registration fees at national meetings
- ✓ Opportunities to participate in online communities and share information and ideas with colleagues.

Yes - all of this for only $36 per year or less than 10 cents per day!

Wait, there is still more – NJAAPT will add a 1 year NJAAPT Section Membership to all new AAPT Regular Memberships or new Associate Memberships that take place between September 1, 2009 and December 31, 2009. If you are already a NJAAPT section member, one additional year will be added to your membership.

(Please note to be eligible for the 1 year you must indicate that you are in the "**NEW JERSEY**" section in the appropriate section of the application. This is vital to ensure that you are correctly credited to the NJ section.

AAPT applications are available online at:

Associate Membership: http://aapt.org/Membership/upload/2009-Associate-Application.pdf

Regular Membership: http://aapt.org/Membership/loader.cfm?csModule=security/getfile&pageid=21848

The American Association of Physics Teachers (AAPT) is the world's leading organization for physics educators with more than 10,000 members worldwide. AAPT provides extensive opportunities for professional development, networking, and student enrichment. AAPT offers many programs, benefits, and services. By joining AAPT, you are also supporting the advancement of physics education. AAPT is as much a membership organization as it is an advocacy movement in physics education.

http://njaapt.org/ **www.aapt.org**

PHYSICS OLYMPICS 2010

We look forward to your school's participation at the NJAAPT sponsored "NJ Physics Olympics" to be held on Saturday, January 16th, 2010 at Monmouth Regional High School. The competition is a lot of fun and your students will meet, share experiences, and compete with other students from various parts of New Jersey. This year's events include: The Fermi Questions, The Cantilever, The Bernoulli Ball, The Catapult, The Windmill and The Spaghetti Gumdrop Tower. The Rules as well as the Registration Form are available on the NJAAPT web site at www.njaapt.org. Last year over 200 students from 25 New Jersey High Schools participated. Please consider joining us as either participants or spectators.

EVENT #1	EVENT #2	EVENT #3	EVENT #4	EVENT #5	EVENT #6
Fermi Question	**Cantilever**	**Bernoulli Ball**	**Catapult**	**Windmill**	**Gumdrop Tower**

NJAAPT Activities Calendar 2009-2010 School Year

Saturday November 14th	**Nuclear Physics Make-n-Take** **Build your own collision simulation apparatus!** **Hillsborough HS - 9:00AM-12:00PM. $25 for NJAAPT members, $35 for non-members including 1 year NJAAPT membership. Click HERE for a downloadable pdf registration form.** **For more information contact Tibi Dragoiu-Luca at** **tdragoiuluca@hillsborough.k12.nj.us**
Friday December 4th	**Holiday Treats at Rutger's University! Physics Lecture Hall.** **Pizza at 6:00 PM - Distribution of boxes 7:00 PM**
Monday January 4th - 2010	**NJAAPT Executive Board Meeting** **Rutgers University - 5:30 PM**
Saturday January 16th	**Physics Olympics at Monmouth Regional High School** **8:00 AM - 2:30 PM** **Click HERE for more information!** **Deadline for all applications January 5th, 2010.**

2009 AAPT Summer Meeting University of Michigan in Ann Arbor

The saga begins; I packed up the Explorer the night before so I was ready to go. I pulled out of my driveway in North Jersey on Saturday morning, August 25th, and 9 hours and 6 minutes later I was parked in front of the Physics building on the campus of the University of Michigan, the host for the 2009 AAPT summer meeting in Ann Arbor. The timing was perfect, as I pulled up, I ran (almost over) into a few long time colleagues who were on their way to dinner, I invited myself in and enjoyed a good meal and a lot of catching up. Then off to my motel to crash and burn till my 1:00 PM Physics of Toys workshop on Sunday.

The toys workshop was loads of fun; we learned a little and played a lot. I picked up a few good ideas to use with my kids and for our Holiday Treats Workshop. Unfortunately I had to leave the “Toys Workshop” a little early to attend the first of several committee meetings, but duty called.

If you have never served on an AAPT National Committee, believe me, it is a very rewarding and a humbling experience. I suggest you try it sometime, or maybe start small and start by attending our NJAAPT executive board meetings; you are always invited and always welcome. I am honored to serve on several AAPT committees as your NJ Section Representative. So at 3:30 PM I attended the “Nominations Committee” meeting where about a dozen of us reviewed the various nominations and prepared our recommendations to the AAPT Executive Board for future officers, committee chairs and committee members.

At 5:30 PM I moved across the hall where met up with our fearless leader Ray Polomski. We both attended the Section Officers Exchange which was immediately followed by the Section Representatives meeting. A great deal was discussed and I won’t go into detail, however a major portion of both meetings revolved around membership and the new “Associate Membership” which will be discussed in a separate part of the newsletter.

Following the Section Representatives meeting was the opening of the exhibit area along with some refreshments. It was here that Ray and I met up with a few more of our New Jersey contingent, David Maiullo, John Roeder and Anthony Lapinski to name a few. As the refreshments ran out, the crowd began to thin out and it was crash and burn time again after a long day.

Monday morning unexpectedly turned out to be one of the highlights of my summer meeting. Standing outside of the hotel, waiting for the shuttle bus was not a scheduled moment to remember but certainly turned out to be one. Standing there along with me and several other people was John Rigden! John is an internationally renowned American physicist and author. I thought it was him, so I quickly flipped open my program guide the check the picture. John was being awarded the AIP’s Andrew W. Gemant Award that afternoon so his picture was prominently displayed in the program. Now having my “IN”, I approached John, introduced myself and congratulated him. John has an incredible resume; he was editor of the American Journal of Physics for over 10 years. At age 92, he served as the Director of Development of the National Science Standards Project at the National Academy of Sciences, Chairman of the History of Physics Forum of the American Physical Society, and has also served on numerous committees for the AAPT, the American Physical Society, the American Association for the Advancement of Science, and the National Academy of the Sciences.

John is also a renowned author. A few of his books include: **Hydrogen: The Essential Element**, **Einstein 1905: The Standard of Greatness**, **Physics and the Sound of Music**, and **Rabi, Scientist and Citizen** to name a few. It was the **Rabi** book that I had read several years ago so I thought I would try to impress him by commenting on some of the points I knew about **Rabi** and the work he did developing RADAR at MIT during WWII. Well, it worked like a charm and John opened up like a like a kid in a candy store. He began telling a few stories and the small crowd of people waiting for the shuttle gathered around to hear. The arrival of the shuttle did not spoil the moment as the discussion continued through several stops and all the way to the Physics building.

A large portion of the discussion involved lending some insight into the book his is currently writing about the relationship between Feynman and Gell-Mann at Caltech. I am not sure when it will be completed, but it sounded like a must read. So lunch in Ann Arbor, 8 bucks, hotel room, 60 bucks, talking physics with John Rigden, priceless!

Coincidentally, also being recognized that afternoon with the AAPT Distinguished Service Award was our very own Dave Maiullo. Dave is best known for the many public physics demo shows he performs in his community, both in the usual locations (libraries, schools, senior centers, science fairs, etc.) and in the unusual (bars, outdoors before rock bands play, street fairs, Coney Island). These efforts led to his recognition with the Ernest E. McMahon Award for Public Outreach from Rutgers University in 2000 and Rutgers University's President's Staff Excellence in Service award in 2006. David plays an invaluable role in the preparation of physics teachers in New Jersey and in the greater physics teaching community. He leads workshops for New Jersey teachers, and his efforts are integral to the NJAAPT and Rutgers relationship. David was the 2006 recipient of the NJAAPT Lifetime Achievement Award. He has served as Past-president and as a member of the Physics Instructional Resource Association (PIRA), an AAPT affiliate, and continues his work at the Summer Meeting with the Lecture Demo Workshop he has led for five years.

Ray Polomski and I both felt obligated to support Dave, so we registered for the Awards Banquet on Wednesday evening to recognize all of the award winners. Well we were there, only problem, no Dave! You see, the banquet was scheduled prior to the "Demo Show" so with his PT Barnum mentality, he passed on the dinner to set up for the "Demo Show". You can catch a glimpse of Dave and the marvelous demo show by clicking AAPT PIRA Demo Show 2009 and PIRA Physics Demo Show.

Well, I seem to have skipped a few things, but I think you can get the picture. I won't go into the details of each session I attended. It was a great meeting; there were some outstanding talks and presentations throughout the week. You can review the entire program (until AAPT archives it) at 2009 Events.

After the AAPT meeting at Ann Arbor I headed north, no, not the long way back to New Jersey, but to East Lansing and a week at Michigan State and the Physics of Atomic Nuclei (PAN) program. See Tibi's article on PAN, and our upcoming workshop involving what we experienced at PAN.

Joe Spaccavento, AAPT New Jersey Section Representative

Physics of the Atomic Nuclei (PAN)

During the summer of 2009, August 2-7, to be more exact, Joseph Spaccavento (North Arlington High School) and Tiberiu Dragoiu-Luca (Hillsborough High school) attended the Physics of the Atomic Nuclei (PAN, www.nscl.msu.edu/teachersstudents/programs/pan) program of the Joint Institute for Nuclear Astrophysics (JINA, www.jinaweb.org/) at National Superconducting Cyclotron Laboratory (NSCL, www.nscl.msu.edu/) on the beautiful campus of Michigan State University (MSU) in East Lansing, MI.

NSCL is a world-leading laboratory for rare isotope research and nuclear science education. It is supported and funded by the U.S. National Science Foundation (NSF). The laboratory operates as a national user facility that serves more than 700 researchers from 100 institutions in 35 countries. A medical cyclotron built by the laboratory in the 1980s was used to treat cancer patients at Harper University Hospital in Detroit for more than 15 years. The technology developed was carried further by medical companies that built higher-powered medical cyclotrons for the treatment of cancer. Over the years, NSCL has evolved into the largest campus-based nuclear science facility in the country. Today, the laboratory has 300 employees, including 28 faculty and about 100 students, half of them in doctoral programs. Recently, MSU and NSCL were selected by Dept of Energy as the site for building, over a decade, the Facility for Rare Isotope Beams (FRIB), an estimated 550 million dollar construction.

Throughout the program, we were introduced to the fascinating fields of astrophysics, cosmology and nuclear science by attending morning lectures presented by NSCL and MSU faculty: “Introduction to Nuclear physics” by Dr. Bradley Sherrill; “Studying exotic nuclei with the Modular Nuclear Array (MoNA)” by Dr. Artemis Spyrou; “MoNA Detecting Physics” by Dr. Thomas Baumann; “Fundamental Symmetries” by Dr. Scott Pratt; “The Accelerating Universe” by Dr. Meghan Donahue; and “How to Make Gold (applying PAN materials in class)” by Dr. Hendrik Schatz. All talks were followed by about one hour of questions and answers. The presenters were very knowledgeable in their field, but also they were not afraid to answer “We don’t know yet!” to a question from the audience. They are performing at NSCL fundamental research, their line of work is still looking for answers and physical explanations to observed phenomena and collected experimental data.   The PAN 2009 resources will be on line soon, for now the PAN 2008 resources are on line at

http://www.nscl.msu.edu/teachersstudents/programs/pan/pan2008.

In the afternoons we had the privilege of working with MoNA. MoNA is used to detect subatomic particles, but it also sensitive to cosmic rays. Each MoNA bar is a plastic scintillator. When struck by a particle, light is emitted and travel to either side of the bar where a photomultiplier is attached. When photons enter the photomultiplier, they hit a photocathode and an electron is emitted. By applying a high voltage, the electron is accelerated towards a set of dynodes. Each time an electron hits a dynode, it frees more electrons that in turn also accelerate and free further electron in the next dynode. At the end, the electrons are collected in an anode where a current is produced that can be measured and registered in a data acquisition (DAQ) system.

All 18 participants to the program (which by the way, were middle and high school teacher from all over the country – California, Massachusetts, Texas, Tennessee, Washington, New Jersey, Michigan, Pennsylvania, Washington state) were split in groups of 3. In the first part of the experiments with MoNA we used an oscilloscope to analyze the signal from the scintillator bar and we learn about some of the properties of the detector. Two important observables that can be used to learn something about an event (e.g. the location of the hit in the scintillator): timing (difference between the arrival times of the signal on each end) and charge (the amount of charge collected on each side). After we familiarized ourselves with the information provided by the oscilloscope, it was time for some quantitative experiment. Dr. Remco Zegers placed a radioactive material at X distance away from one photomultiplier's end. Our job was to calculate the location based on the data read off the oscilloscope's screen. Then, instead of using the oscilloscope we learned about a more powerful investigative tool, the "SpecTcl" analysis software. SpecTcl is used to read data, create spectra and study them. After some practice and familiarization with the the software, again, Dr. Remco placed a radioactive probe somewhere and using SpectTcl we had to find the exact location. Furthermore, we learned how to study some of the main characteristics of muons, such as their velocity and their angular distribution using the data provided by the SpecTcl software.

After one week, the program ended with poster presentations. Each group worked on a poster summarizing the work performed during the program. (posters will be online shortly)

In two evenings we mingled with a group of physics teachers from South Korea, participants in the Visiting International Professional Program (VIPP, http://www.isp.msu.edu/VIPP/about/glance.htm).

In another evening we visited the Impression 5 Science Center, http://impression5.org/index.php . It is a hands-on learning environment that challenges the visitors to experience, discover and explore the world around us using our five senses. If you are ever traveling through East Lansing, MI it's worth spending about 2-3 hours visiting the Impression 5 Science Center.

For information on learning nuclear science with marbles, visit the web page http://www.jinaweb.org/outreach/marble/. There are lessons and hands-on activities, including teacher's guide, and games with nuclear physics (Isotope Bingo, Nucleosynthesis with dice, and the fragmentation box). "Playing" with the fragmentation box was a lot of fun, it is a device that simulates a nuclear collision. Upon return, we decided to organize a "Make-and-Take" workshop for the physics teachers in New Jersey. It is scheduled for Saturday, Nov 14 at Hillsborough High School between 9am-1pm.

We recommend the PAN program; visit http://www.nscl.msu.edu/teachersstudents/programs/pan for information about how to apply next year. If you have any questions please email or call, we have promised that New Jersey will be represented again next summer.

Tibi Dragoiu-Luca & Joe Spaccavento

Membership Renewals

The Section executive board has discussed a growing problem – that of declining current membership. At present we have but 181 currently paid members on our roll – that is, members who have paid their membership fee for the year 2009 and beyond. We are carrying far more than that number, people who have not paid their dues and yet reap the benefits of the organization.

If you have not paid your dues for this year or any previous year, please consider doing so as soon as possible. It is our intention to keep the section as financially viable as possible and to do so requires funding from the collection of dues. The $10/year or $25/3 years has not been raised since the organization of the New Jersey Section.

If you are not certain of whether or not you are current in our Section, email Dave Bandel or Jim Kovalcin for that information. A membership renewal form is available at our website and the dues should be sent to Dave Bandel. Please take the time to determine your status and, if you need to re-register.

Physics "C" Corner

My intention, over the next few of issues of the NJAAPT newsletter, will be to help develop those topics which are necessary supplements for converting a "B" level physics course into a calculus based "C" level course. Particularly at the Mechanics C level, the changes are relatively few while at the Electricity & Magnetism level the changes are much more extensive.

This month I will continue with use of calculus in the determination of the moment of inertia of a disk about its axis of symmetry and perpendicular to the plane of the disk through integration.

Due to length, this article will only be included in the electronic version of the newsletter.

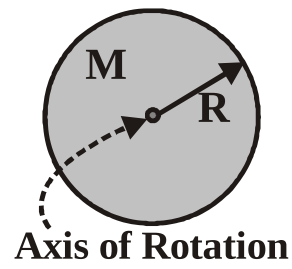
Feedback is welcome and should be directed to JimTHX@comcast.net.

Additional Physics C resources can be found on my website at www.JimTHX.vze.com.

I certainly hope that these articles will useful particularly to Physics C Teachers, but to all other physics teachers as well.

Jim Kovalcin, Manalapan High School

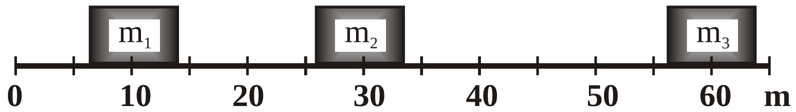
Calculating Moment of Inertia - Integration



Suppose that you would like to determine the Moment of Inertia of a uniform disk, which has a mass of M = 12.0 kg and a radius of R = 15.0cm, about an axis directed through and perpendicular to the center of the disk as shown to the right.

Before we begin this problem, let's first review the discrete problem!

Consider a system consisting of three discrete mass arranged as shown below. Where m_1 = 35kg, m_2 = 54kg and m_3 = 82kg.



The moment of inertia of this system about x = 0m can be determined by using:

$I = m_1 \cdot x_1^2 + m_2 \cdot x_2^2 + m_3 \cdot x_3^2 + m_n \cdot x_n^2$

Which in mathematical notation can be written as:

$$I = \sum_{n=1}^{3} \left(m_n \cdot x_n^2 \right)$$

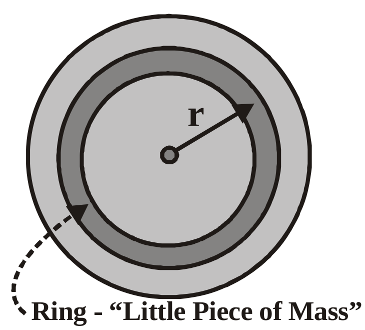
The moment of inertia I n this case this becomes:

$I_{x=0} = 35 \cdot 10^2 + 54 \cdot 30^2 + 82 \cdot 60^2 = 3.47 \times 10^5\ kg \cdot m^2$ **[Moment of inertia I of the system]**

Now using this same approach let's determine the moment of inertia of the solid disk. But in order to do this we have to first solve a problem. Since the mass of the disk is not all the same distance from the center of rotation we cannot simply multiply the mass of the disk **M** by the radius of the disk **R** squared.

So what is the solution?

What if we broke the disk up into little pieces such that all the mass in each little piece is the same distance from the center of rotation?



If that were true, we could then determine the moment of inertia of each little piece **ΔI** by multiplying the mass of that little piece **Δm** by the distance of that little piece from the center of rotation squared $\mathbf{r^2}$. [$\Delta I = r^2 \cdot \Delta m$]

The question is what would the shape of that little piece have to be so that all the mass in that little piece is approximately the same distance from the center rotation?

The answer is a ring! Notice in the ring at the right that all the mass in this ring is approximately the same distance **r** from the center of the disk, the center of rotation.

So, if we knew the mass of that ring **Δm**, and since all the mass in that ring is approximately the same distance from the center of rotation **r**, we could multiply the mass of that ring **Δm** by the average distance of that mass from the center of rotation squared $\mathbf{r^2}$ to find the moment of inertia of that ring $\Delta I_{ring} = r^2 \cdot \Delta m$.

The next step will be to determine the mass Δm of this ring.

First of all, we can make a pretty good approximation of the mass of this ring by calculating the average mass density of the disk **σ** and then multiply that average mass density by the area **ΔA** of the ring.

The average mass density of the disk **σ** can be found by dividing the total mass **M** of the disk by the total area $\mathbf{A_{disk}}$ of the disk.

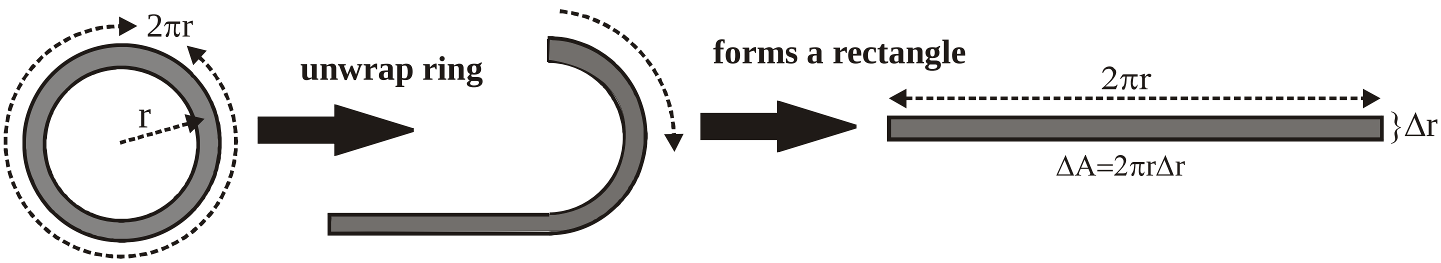
$$\sigma = M/A_{disk}$$

In the case of a disk the area will be $\mathbf{A_{disk} = \pi \cdot R^2}$ and so the mass density per unit area s becomes:

$$\sigma = M/(\pi \cdot R^2)$$

But what is the area ΔA of the ring?

As hard as it may initially seem to believe, we can get a pretty good approximation of the area **ΔA** of this ring by treating the ring as if it were a rectangle. Imagine taking this ring and "unrolling" it into a rectangle as shown below.



[You might argue that this rectangle does not really correspond to the area **ΔA** of the ring, but if the thickness of the ring **Δr** is **VERY** small, then the inner circumference of the ring will be almost the same length as the outer circumference of the ring, in which case it will be quite appropriate to use the area of the rectangle as a very good approximation of the area of the ring. Ultimately, if we allow the thinness of the ring **Δr** to approach zero, the area of the rectangle and the area of the ring will be exactly the same!]

Since the area of a rectangle is equal to the length **L** multiplied by the width **W**, $\mathbf{A_{rectangle} = L \cdot W}$, the area of the ring **ΔA** will be:

$$\Delta A = L \cdot W = 2 \cdot \pi \cdot r \cdot \Delta r$$

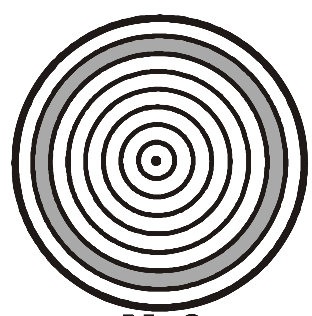
Now that we know the area of the ring **ΔA,** we can now determine the mass of this ring since the mass of the ring will be equal to the product of the mass density of the disk **σ** multiplied by the area **ΔA** of the ring:

$$\Delta m = \sigma \cdot \Delta A = \sigma \cdot [2 \cdot \pi \cdot r \cdot \Delta r] = 2 \cdot \pi \cdot \sigma \cdot r \cdot \Delta r$$

The moment of inertia of this ring **ΔI** can now be determined by multiplying the mass of this ring **Δm** by the distance of this mass from the center of rotation squared $\mathbf{r^2}$.

$$\Delta I = \Delta m \cdot r^2 = 2 \cdot \pi \cdot \sigma \cdot r \cdot \Delta r \cdot r^2 = 2 \cdot \pi \cdot \sigma \cdot r^3 \cdot \Delta r$$

Now that we know the moment of inertia of one ring **ΔI** we can, in principle, determine the moment of inertia $\mathbf{I_{disk}}$ of the entire disk by breaking the disk up into many rings as seen to the right, determine the moment of inertia of each ring and then add these individual moments of inertia together to determine the total moment of inertia of the disk $\mathbf{I_{disk}}$.



N=8

$$I_{disk} = \Delta m_1 \cdot r_1^2 + \Delta m_2 \cdot r_2^2 + \Delta m_3 \cdot r_3^2 + \ldots = \sum_{n=1}^{8} \left(\Delta m_n \cdot r_n^2 \right)$$

If we now take the limit of this sum as the number of rings goes to infinity, we will get rid of any error introduced by our approximation of the area of the ring as being the same as the area of a rectangle and we will, likewise, eliminate any error caused by some mass in the ring being closer or farther from the center of rotation.

$$I_{disk} = \lim_{N \to \infty} \sum_{n=1}^{N} \left(\Delta m_n \cdot r_n^2 \right)$$

Taking the limit of this sum is, of course, the same thing as taking the integral of $\mathbf{r^2 \cdot \Delta m}$!

$$I_{disk} = \lim_{N \to \infty} \sum_{n=1}^{N} \left(\Delta m_n \cdot r_n^2 \right) = \int_0^R r^2 \, dm$$

Remember from above that the mass of each individual ring is given by:

$$\mathbf{\Delta m = \sigma \cdot \Delta A = \sigma \cdot [2 \cdot \pi \cdot r \cdot \Delta r] = 2 \cdot \pi \cdot \sigma \cdot r \cdot \Delta r}$$

As we allow the number of rings to approach infinity the differential $\mathbf{\Delta m}$ becomes **dm**, $\mathbf{\Delta A}$ becomes **dA**, while the differential $\mathbf{\Delta r}$ becomes **dr**.

$$\mathbf{dm = \sigma \cdot dA = \sigma \cdot [2 \cdot \pi \cdot r \cdot dr] = 2 \cdot \pi \cdot \sigma \cdot r \cdot dr}$$

Therefore, the moment of inertia of the disk $\mathbf{I_{disk}}$ becomes:

$$I_{disk} = \int_0^R r^2 \, dm = \int_0^R r^2 \cdot (2 \cdot \pi \cdot \sigma \cdot r) \, dr = 2 \cdot \pi \cdot \sigma \cdot \int_0^R r^3 \, dr$$

$$I_{disk} = 2 \cdot \pi \cdot \sigma \cdot \int_0^R r^3 \, dr = 2 \cdot \pi \cdot \sigma \cdot \frac{r^4}{4} = \frac{1}{2} \cdot \pi \, \sigma \cdot R^4 - \frac{1}{2} \cdot \pi \cdot \sigma \cdot 0^4 = \frac{1}{2} \cdot \pi \, \sigma \cdot R^4$$

But the total mass **M** of the disk is equal to the mass density $\boldsymbol{\sigma}$ of the disk multiplied by the total area $\mathbf{A = \pi \cdot R^2}$ of the disk.

$$\mathbf{M = \sigma \cdot A = \sigma \cdot (\pi \cdot R^2) = \pi \cdot \sigma \cdot R^2}$$

Therefore, the moment of inertia of the disk will be:

$$\mathbf{I_{disk} = ½ \cdot \pi \cdot \sigma \cdot R^4 = ½ \cdot [\pi \cdot \sigma \cdot R^2] \cdot R^2 = ½ \cdot M \cdot R^2}$$

Remembering from the beginning of this discussion that our disk has a mass of M = 12·kg and a radius of R = 0.15·m, the moment of inertia of this uniform solid disk $\mathbf{I_{disk}}$ becomes:

$$\mathbf{I_{disk} = ½ \cdot 12 \cdot 0.15^2 = 0.135 kg \cdot m^2}$$