Five Laws of Physics in Five Seconds of Data Collection for a cost of Five Dollars

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Purpose: verify five principles of physics in five seconds of data collection for five dollars

Apparatus: Vernier force sensor, Vernier motion sensor, Logger Pro, ring stand and clamp
spring and pool ball ($5)   (motion sensor is below ball in picture)

Description
A Vernier force meter is attached to a rod on a ring stand. A small steel spring $2 is hung from the force meter, with a pool ball ($3) attached to the bottom end of the spring. A motion sensor is placed on the table facing upwards at the pool ball. The system is allowed to come to equilibrium. At this moment, the force acting on the force meter is the static weight of the spring and the pool ball. Zero the force meter and the motion sensor using Logger Pro's zero sensor capability. Give the ball a push up/down and the ball oscillates in simple harmonic motion. The force meter will measure the force acting on the top of the spring and the motion sensor will record the position of the ball. Five second of data are collected at a sample rate of 20 Hz. Logger Pro will calculate the velocity and acceleration of the ball and the elastic potential energy of the spring and the kinetic energy of the ball, all in real-time.

Goals: To verify the following basic Physics Principles in 5 seconds of data collection

Velocity is the derivative (slope) of position-time graph, and acceleration is the derivative (slope) of velocity-time graph, as the ball bounces up/down in simple harmonic motion.

Hooke's Law: the spring stretch is directly proportional to the net force acting on the spring as the ball is oscillating up and down. $\mathbf{F = -k X}$

Newton's Second Law: the acceleration of any object of fixed mass is directly proportional to the net force acting on that object and is in the same direction as that net force.
Also, the acceleration is inversely proportional to the mass, if the force is constant.

$$\mathbf{A = F / M}$$

Mechanical Energy is conserved if no external work is done on the ball-spring system.
The sum of the elastic potential energy of the spring and kinetic energy of the ball is constant.

KE (ball and spring) + elastic PE (spring) = constant

In simple harmonic motion (SHM) the period of an object depends only on the mass (inertia) **m** of that object and on the restoring spring constant **k**. The period (**T**) is not a function of the acceleration of gravity, g, or the weight of the system, mg.

$$\mathbf{T = 2\pi \sqrt{m/k}}$$

With the <u>calculated</u> values for "**m**" and "**k**" from our graphs, we should be able to predict the period of the system.

Procedure:
Start Logger Pro.

Set the DATA COLLECTION for 20 samples/second for 5 seconds.
Thus, we will have a total of 5 seconds and 100 data samples to verify five laws of physics.

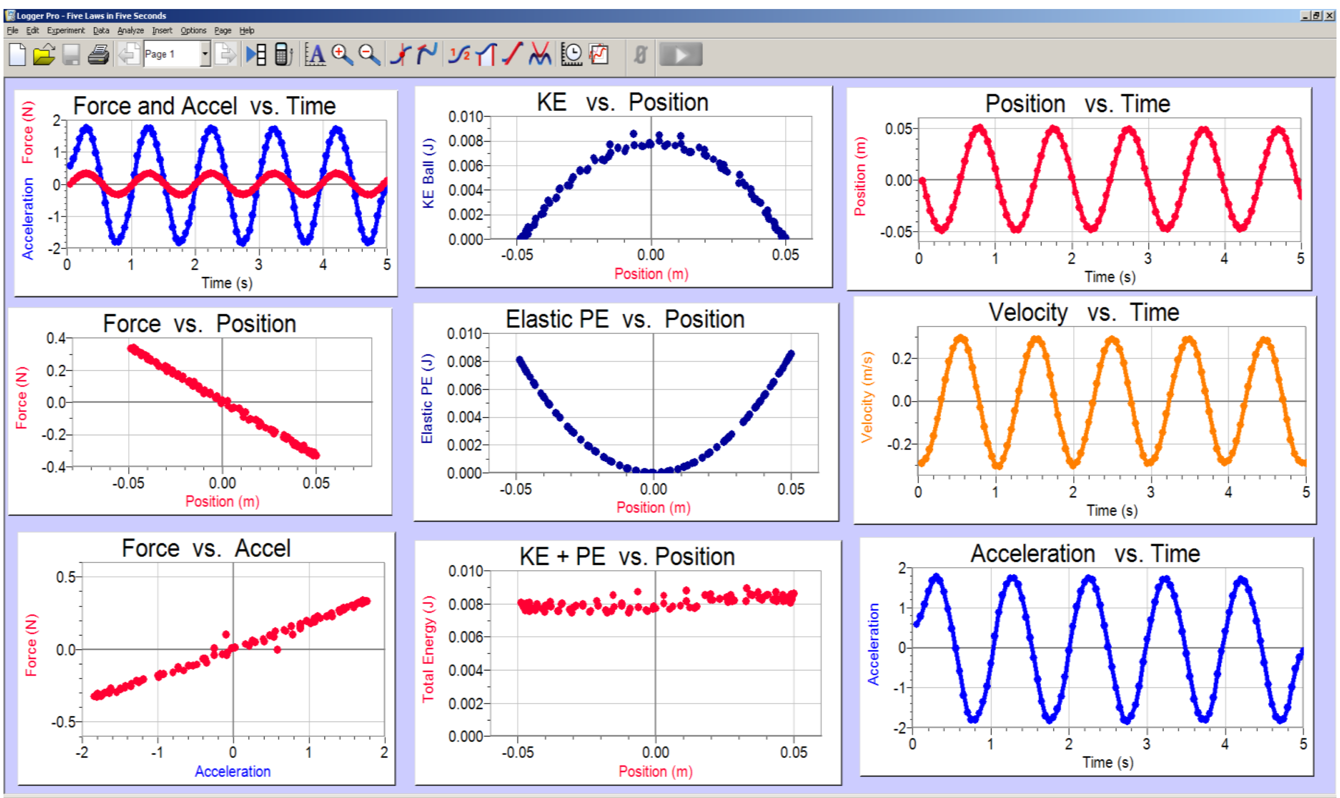
Be sure to **ZERO** both force meter and motion sensor when the ball is **<u>at rest</u>** at its equilibrium position. The ZERO button is on the top MENU.

Then give the ball a slight push up or down..... not too big a push.
You want smooth oscillations, not a ragged bobbling.

Wait a few seconds, then press the **COLLECT** button.
The system will record data, and plot 9 different graphs.

Examine each of the graphs.

Can you verify each of the goals with the graphs?



CAN YOU DEMONSTRATE the following:

The maximum positive velocity occurs where the SLOPE of the position-time graph has its greatest positive value. The maximum negative velocity occurs where the slope of the position-time graph has its maximum negative value. Where the slope of the position graph is zero, the velocity is zero.

The velocity is the slope (derivative) of the position-time graph.

The maximum positive acceleration occurs where the SLOPE of the velocity graph has its greatest positive value. The maximum negative acceleration occurs where the slope of the velocity graph has is greatest negative value. Where the slope of the velocity graph is zero, the acceleration is zero.

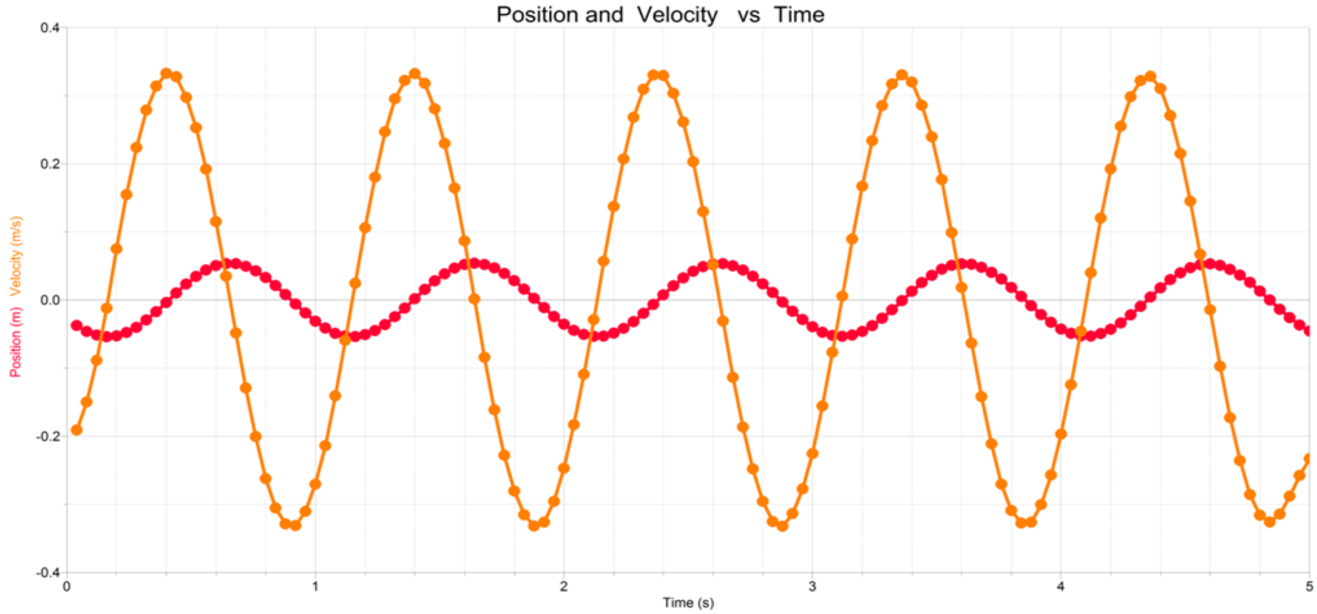
The acceleration is the slope (derivative) of the velocity-time graph.

The **maximum positive acceleration** occurs when the spring is **longest** and the **maximum negative acceleration** occurs then the spring is **shortest**.

The **maximum velocity** occurs when the spring is at its equilibrium position, in the <u>middle</u> of each bounce. At this location, the acceleration is zero.

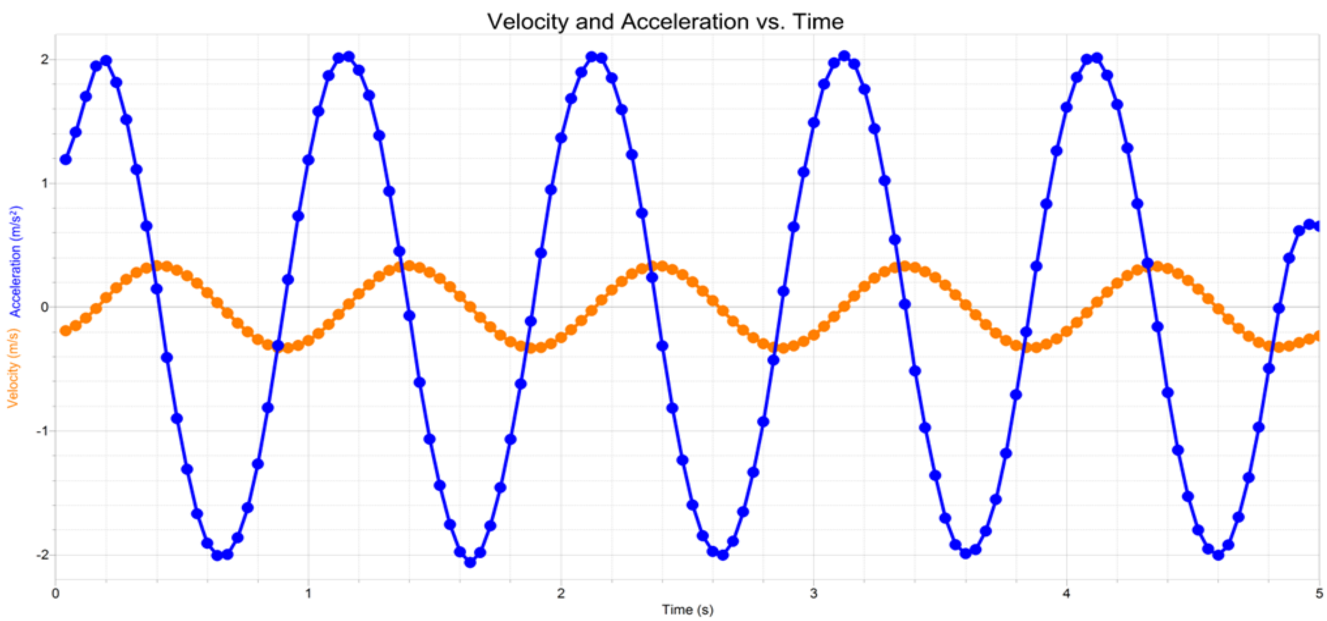
Position (red) and Velocity (orange) vs. Time

When the slope of the position-time graph is maximum, is the velocity maximum?
When the slope of the position-time graph is minimum, is the velocity minimum?
When the slope of the position-time graph is zero, is the velocity zero?



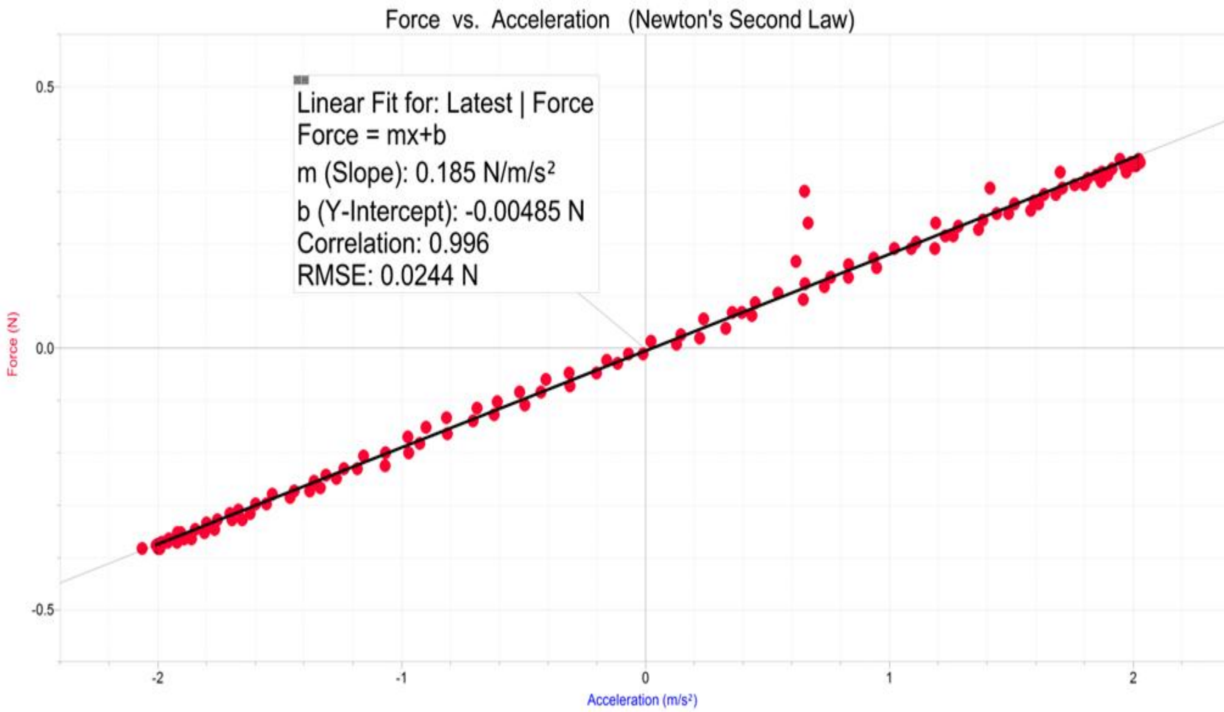
Velocity (orange) and Acceleration (blue) vs. Time

Does the maximum acceleration occur when the slope of the velocity graph is maximum?
Does the minimum acceleration occur when the slope of the velocity graph is minimum?
When the slope of the velocity-time graph is zero, is the acceleration zero?

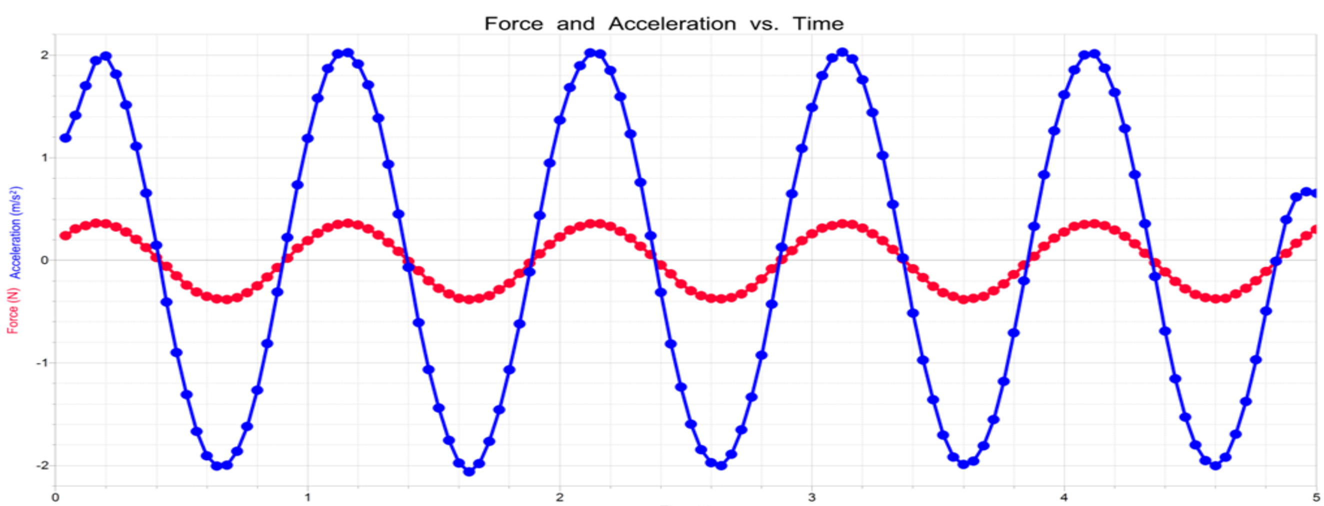


Newton's Second Law states that the acceleration of any object (of fixed mass) is “directly proportional to the applied net force”.
Does your force-acceleration graph verify a direct proportion?
Is the graph LINEAR ? Does the graph pass through the origin (0,0)?

Do a linear curve fit to determine the slope. The slope is the effective inertial mass, in kg.
Slope = 0.185 kg



The **FORCE** and the **ACCELERATION** vs. **TIME** graphs are both **<u>sine waves</u>**, with the maximum and the minimum force and acceleration happening simultaneously. When the force is zero, the acceleration is also zero.

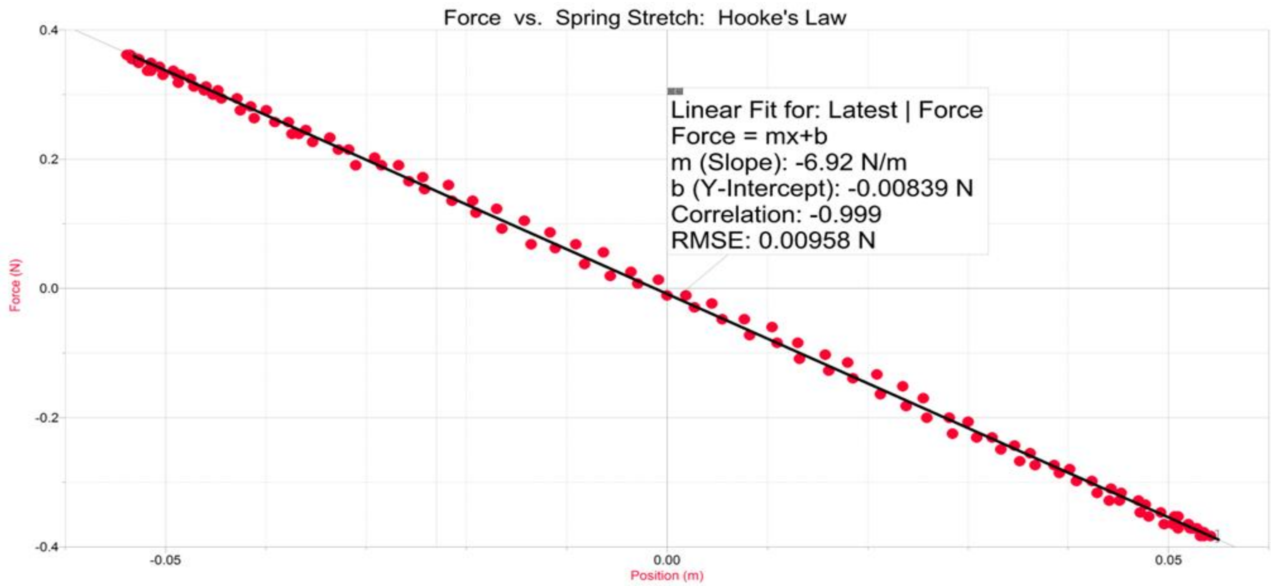


The **FORCE** vs. **POSITION** graph is LINEAR with a negative slope and passes through the origin (0,0). Because the position sensor is on the table, facing upwards towards the bottom of the pool ball, when the force is greatest (spring is longest) the distance from the ball to the sensor is minimum. This accounts for the negative slope.

This verifies what is called **Hooke's Law: $F = -kX$**
where F = force acting on spring (N) X = spring stretch (m) and k = the spring constant

Determine the spring constant, k.

Expand this graph and do a "linear curve fit".
Calculate the SLOPE of the force-position graph to determine the value of the spring constant, k. **k = - 6.92 N/m**

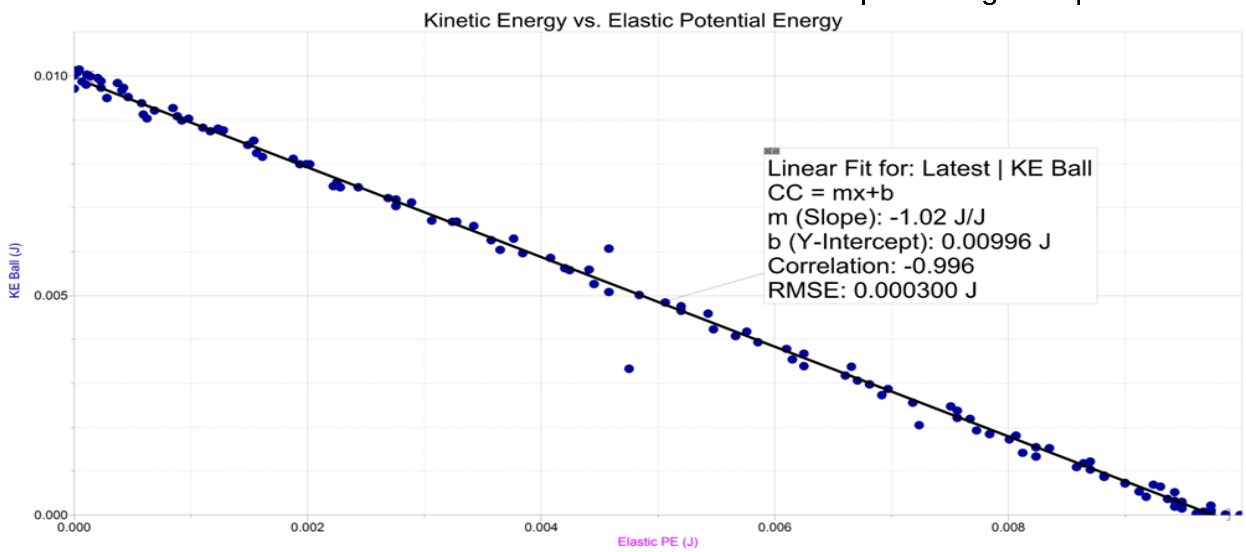


Often times the spring constant is determined by hanging known masses (known weights) from a spring and measuring the length of the spring with a ruler. Then one can plot force (weight hanging from spring) vs. spring length and test if the graph is "linear". That process probably takes 20 minutes or more, and uses perhaps 5 to 10 data pairs. This method using the bouncing ball on spring takes five seconds, and uses 100 data pairs to plot the graph and calculate the slope, the spring constant. The force precision is +/- 0.01 N and the position precision is +/- 0.001 m.

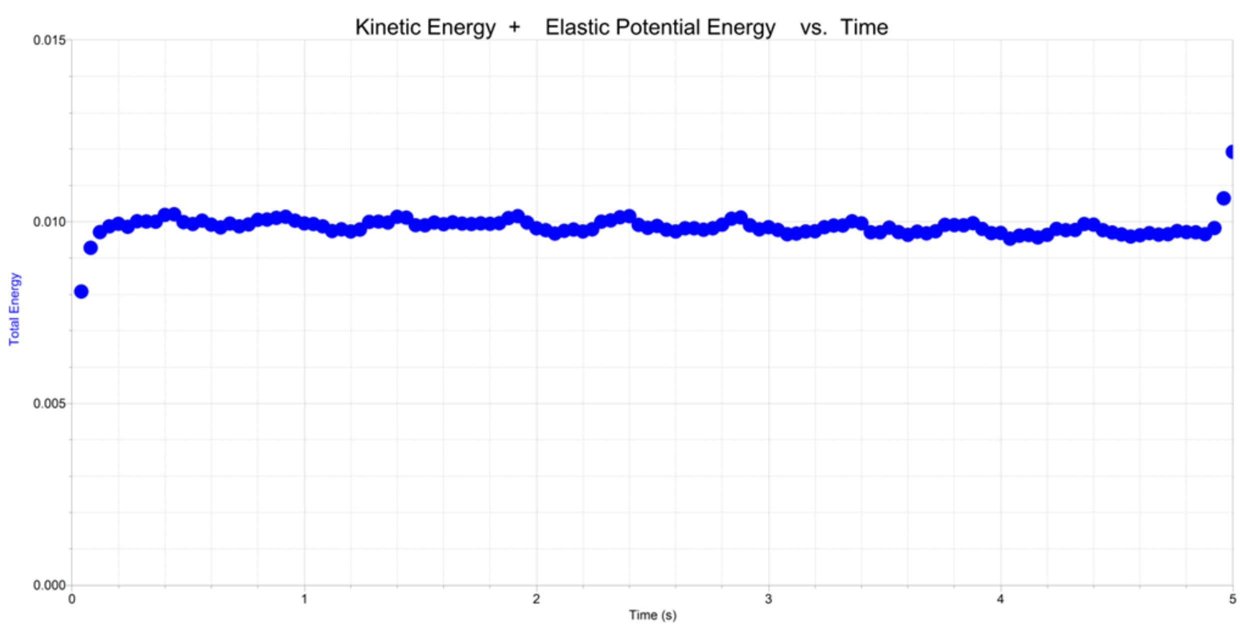
Conservation of Energy
Test whether the bouncing ball and spring represents an example of the **Conservation of Energy**. Is the sum of the KE of the ball and spring plus the elastic potential energy of the spring a constant value?

If true, what should be the shape of the graph of KE vs. elastic PE ?

Kinetic Energy vs. Elastic Potential Energy
If KE + PE = constant then KE = constant - 1.0 PE predicting a slope of -1.0



If you add the PE (spring) plus the KE (ball) do you see that the **sum** is constant?
Does your graph verify the Law of Conservation of Energy?
Here is the **sum** of the KE and Elastic PE plotted against time = reasonably constant



Simple Harmonic Motion (SHM)
The case of a bouncing mass at the end of a spring represents a kind of motion referred to as "simple harmonic motion" where the restoring force in the spring (F) is directly proportional to the stretch of the spring: **F = - kX**

The **resonant period** (time for one bounce) of an oscillating spring and mass is related to how much mass (inertia) is moving, and how stiff the spring is.
The resonant period (T) is governed by the equation: **T (seconds) = 2 ∏ √(m/k)**

You can determine the effective mass **m** of the system of spring-ball from the slope of the force-acceleration graph: since F = m A The slope is 0.185 on the graph so the effective mass of the ball-spring system is 0.185 kg. **m = 0.185 kg**

You can determine the spring constant (**k**) from the slope of the force-position graph
F = - k X **k = - 6.92** N/m

Using your values of the mass of the oscillating system **m** (in kg) and the value for your spring constant (**k**) determine the predicted period (T) of oscillation of the pool ball-spring system. **T = 2 π √m/k = 2 π** √ (0.185/6.92) = **1.02 seconds**

Use the EXAMINE button on the MENU and look at the peak times for a few oscillations. Calculate the time between peaks (the period). **T = 1.64 – 1.00 = 1.00 seconds**

Does the measured value for the period agree with the predicted period based on SHM?